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LONGSTAFF-SCHWARTZ MODEL AND FACILITY PRICING IN MANAGEMENT – FINANCIAL ANALYSIS AND DATA SOLUTIONS

Summary. The ability to adequately select instruments used in financial analysis plays an important role in the management process. The Longstaff-Schwartz model has been created taking into account option spreads with their present and identifiable risks. This paper provides a review of the literature, historical development of key concepts and data analysis which are based on them.

Keywords: Longstaff-Schwartz Model, management, financial analysis, options

MODEL LONGSTAFFA-SCHWARTZA I JEGO WYCENA W ZARZĄDZANIU – ANALIZY FINANSOWE I ANALIZY DANYCH

Streszczenie. Istotne znaczenie w procesie zarządzania odgrywa umiejętność adekwatnego dobrania instrumentów wykorzystywanych w analizie finansowej. Model Longstaffa-Schwartza został stworzony z myślą o spreadach opcji, uwzględniając występujące, identyfikowalne ryzyko. W opracowaniu zaprezentowano badania literaturowe, zmiany historyczne podstawowych koncepcji oraz analizy, które na nich bazują.

Słowa kluczowe: Model Longstaffa-Schwartza, zarządzanie, analiza finansnowa, opcje

1. Introduction

Longstaff and Schwartz[1] derive a simple closed form solution to value credit spread options and assume that the logarithm of the credit spread, on which the option is based, follows a mean reverting process. What is more, a riskless rate is assumed to follow Vasicek's[2] assumptions. The authors use monthly observation for the credit spread between highly graded Moody's bond indices and long-term U.S. Treasury bond yields over the years 1977-1992. Longstaff and Schwartz define the value of a European credit spread call option as C(X, r, T), where X is the logarithm of the credit spread and K is the strike level of the spread.

Additionally the payoff function is:

$$H(X) = \max(0, e^{x} - K), \tag{1}$$

After a number of transformations one obtains the closed form solution for the value of the European call option credit spread:

$$C(X,r,T) = D(r,T)e^{\mu+\frac{n^2}{2}} N(d_1) - KD(r,T)N(d_2), \tag{2}$$

where N( ) stands for the standard normal distribution function with mean μ and variance η2 and D(r, T) is the riskless discount bond with maturity T.

2. Implications of the Longstaff and Schwartz (L-S) Model

The authors noticed that due to the mean reversion of the log spread and riskless rate, the model possesses interesting properties. The Value of the call option can be smaller than its intrinsic value, if the option is in-the-money. Once the credit spread is above the long run mean, it is expected to approach its mean value, and hence, decline. So, in-the-money call options are less likely to remain in this state over time and the value of the option can be below it φnsic value because of the expected decrease in the credit spread. Longstaff and Schwartz[3] note that this fact is contrary to the Black and Scholes[4] model where the underlying asset appreciates following the riskless rate in the risk neutral valuation framework. What is more, with time to maturities ranging from 0.5 to 1.5 years it was found that there might

[1] Longstaff A., Schwartz E.: Valuing Credit Derivatives. „Journal of Fixed Income", 1995.
[2] Vasicek O.: An equilibrium characterization of the term structure. „Journal of Financial Economics", No. 5, 1977, p. 177-88.
[3] Longstaff A., Schwartz E.: op.cit.
[4] Black F., Scholes M.: The pricing of options on corporate liabilities. „Journal of Political Economy", No. 81(2), 1973.

appear a negative convexity when it comes to the price of the call option due to mean reversion. Longstaff and Schwartz found an interesting implication of the mean reverting motion on Delta. They state that similarly to the Black-Scholes formula, Delta on a call spread option remains positive. However, as the time to maturity goes to infinity, Delta reduces its value to zero. The authors also noted that Delta is decreasing as a function of credit spread, meaning the function becomes convex from concave as time to maturity increases. L-S model implies that a long-term (over 12 months) option written on the credit spread may not be a good hedging instrument. Moreover, they find that for some options, price change is more sensitive to changes in the underlying when being out-of-the-money than in-the-money. Following is the Longstaff and Schwartz mean reverting processes of the log spread and risk-free rate are of the form shown in the fig. 1.

$dX = (a - bX)dt + sdZ_1$		**$Dr = (\alpha - \beta r)dt + \varphi dZ_2$**	
dX	**Change in the natural logarithm of spread level**	**dr**	**Change in risk-free rate**
a	**Equilibrium parameter**	**α**	**Equilibrium parameter**
b	**Spread of adjustment**	**β**	**Spread of adjustment**
S	**Volatility of the logarithmic spread**	**φ**	**Volatility of the short-term interest ratio**
dt	**Time period when change commences**		
dZ_1, dZ_2	**Wiener processes of the log spread and risk-free ratio rate respectively**		

Fig. 1. Parameters describing the mean reverting processes of the log spread and risk-free rate
Rys. 1. Parametry opisujące średnią procesu powracania log spread i stopy wolnej od ryzyka
Source: own study.

It should be mentioned that two methods of parameter estimation will be used: Ordinary Least Squares (OLS) and Maximum Likelihood (ML). There is a difference in notation in the mean reverting process equations between the original representations made by Ornstein-Uhlbeck and the one depicted in the previous data. It originates from the multiplication of the term $b(X - X)$ from the former approach, leading to the formula: $(bX - bX)$. This, in turn is equivalent to $(a - bX)$, as $a = bX$. The log spread mean reverting process coefficients, $_0\gamma$ and $_1\gamma$, are estimated with the use of regression. Coefficients X and b are obtained from formulas of the form shown (Dixit and Pindyck[5]).

[5] Dixit A.K., Pindyck R.S.: Investment under Uncertainty. Princeton University Press, New Jersey 1994.

Riskless rate r_t mean reverting process coefficients will be estimated after running the following OLS regression:

$$\Delta r_{t+1} = \psi_0 + \psi_1 r_1 + \varepsilon_{t,r} \,, \tag{3}$$

where, r_{t+1} – is the daily change in the risk-free rate; r_t – is the riskless interest rate of the previous day. It should be stressed that the remaining procedure and coefficient estimation is analogical to the log spread coefficient estimation and is done with the use of:

$$\bar{r} = -\frac{\hat{\psi}_0}{\hat{\psi}_1} \,, \tag{4}$$

$$\hat{\alpha} = \beta \bar{r} \,, \tag{5}$$

$$\beta = -1n(1+\psi_1) \,, \tag{6}$$

$$\varphi_{\varepsilon,r} \sqrt{\frac{1n(1+\psi_1)}{(1+\psi_1)^2 - 1}} \,, \tag{7}$$

where $\phi_{\varepsilon,r}$ (also notated as $\sigma_{\varepsilon,r}$) corresponds to the standard deviation of errors from equation above. All estimations are made with the use of Eviews 5 software.

This method also uses discrete data to estimate coefficients of the mean reverting processes. The Maximum Likelihood method consists of fitting a mathematical model to historical data. In other words, the maximum likelihood function is responsible for finding the coefficients of the given equation by maximizing the likelihood of their occurrence. The ML estimation is done for both the log spread and riskless rate with the use of Matlab v 7.5.

Having estimated the mentioned parameters, one has to calculate the correlation coefficient between the two Wiener processes dZ_1 and dZ_2, as follows from Longstaff and Schwartz. The Correlation coefficient indicates the strength and direction of the relationship between both processes and is of the form:

$$\rho = \frac{\frac{1}{n}\sum_{i=1}^{n}(dZ_1 - \mu_{dZ_1})(dZ_2 - \mu_{dZ_2})}{\sigma_{dZ_1}\sigma_{dZ_2}} \,, \tag{8}$$

where dZ_1 and dZ_2 are obtained after rearranging equation 8 respectively,

Moreover $-1 \le \rho \le 1$.

Bearing in mind the assumptions of the model, the L-S approach allows for the possibility that the interest rate risk and changes in spread are priced by the market: hence, the market risk premiums are incorporated in a and α terms. With this framework, the value of the European call written on the credit spread F(X, r, T), with payoff function H(X) and expiration time T, must solve the following Partial Differential Equation (PDE), according to Longstaff and Schwartz:

$$\frac{s^2 F_{xx}}{2} + \rho\sigma s F_{Xr} + \frac{\sigma^2}{2} F_{rr} + (\alpha - \beta r)F_r + (a - bX)F_X - rF - F_T = 0, \tag{9}$$

where the initial condition is $F(X, r, 0) = H(X)$

The closed form solution for the value of the call option is as follows:

$$C(X,r,T) = D(r,T)e^{\mu+\frac{\eta^2}{2}} N(d_1) - KD(r,T)N(d_2), \tag{10}$$

where N(d) stands for standard normal distribution function. Moreover, X is conditionally normally distributed with mean μ and variance η^2 given by the following equations:

$$\mu = Xe^{-bT_D} + \frac{1}{b}(a - \frac{\rho\sigma s}{\beta})(1 - e^{-bT_D}) + \frac{\rho\sigma s}{\beta(b+\beta)}(1 - e^{-(b+\beta)T_D}), \tag{11}$$

$$\eta^2 = \frac{s^2(1 - e^{-2bT_D})}{2b}, \tag{12}$$

Additionally the payoff function for the call option buyer is:

$$H(X) = \max(), e^x - K), \tag{13}$$

$$d_1 - \frac{-1nK + \mu + \eta^2}{\eta}, \tag{14}$$

$$d_2 = d_1 - \eta, \tag{15}$$

It should be mentioned that $D(r, T)$ is the price of the risk-free discount bond, or in other words the discount factor at which the option is discounted to its present value, and equals e^{-rT}; where r is the riskless rate (yearly basis), here assumed equal to the 3-month LIBOR rate from the last day of the estimation period. As follows, (Hull[6]) one can define:

$$T_D = trading \cdot days \cdot until \cdot maturity, \tag{16}$$

$$T = \frac{calendar \cdot days \cdot until \cdot maturity}{calendar \cdot days \cdot per \cdot year}. \tag{17}$$

Both T and T_D are equivalent and denote the same period of time. The difference between these formulas comes from the fact that the risk-free rate is compounded on a yearly basis and the option is counted on a trading day basis. Considering the number of trading days in

[6] Hull J.C.: Options, Futures and Other Derivatives. Prentice Hall, New Jersey 2002.

a year, as stated by Hull equals 16-17. One has to remember, however, that the number of trading days should be calculated separately for each year, to produce accurate results.

Mun implies that sensitivity analysis shows the outcome of the option price change with respect to the unit change of the underlying variable. Due to the complexity of the pricing model, most of the sensitivity measures are hard to estimate. However, following Longstaff and Schwartz, it is possible to obtain the Delta. The sensitivity of the option with respect to other coefficients will be performed with the use of graphs.

This is an important parameter used for option hedging. As mentioned by Hull, Delta is the rate of change of the price of the option with respect to change in the underlying. Following the author, it is the number of units of stock (when stock is the underlying) that should be kept for each short position in an option in order to create a riskless hedge (the whole process is also called Delta Hedging). Delta (Δ) can be depicted as a slope of the function-relating price of the option to the underlying price.

$$\Delta = \frac{\delta C}{\delta X}, \tag{18}$$

where, C is the price of the call option, and X is the underlying.

As implied by Hull, it can be shown that for the European call option following GBM, written on non dividend paying stock Delta equals:

$$\Delta_{B-S} = N(d_1), \tag{19}$$

where, $N(d_1)$ is related in this case to Equation (2).

Delta hedging, when buying the European call option, involves maintaining a short position equal to $N(d_1)$ in the underlying asset. In the case of the L-S approach, since the formula for the option price is different than the original Black-Scholes[7], the Delta will be different as well.

Longstaff and Schwartz found that after differentiating according to equation 20**,** one obtains the closed form solution for Delta in L-S approach.

$$\Delta = D(t,T)e^{\mu+\frac{\eta^2}{2}} N(d_1) * e^{-bT} * e^{-X}. \tag{20}$$

[7] Black F., Scholes M.: The pricing of options on corporate liabilities. „Journal of Political Economy", No. 81(2), 1973.

3. Results Obtained After Applying the Model

A cursory look at the log TED spread series (fig. 2) suggests that it follows a mean reverting process. It should be stressed that it is especially well visible when looking at time periods like those of years 1997-2001 and 2005-2007 (including the estimation period). Even during the time of the "credit crunch", data seem to be mean reverting, although being slightly above the 10-year TED spread mean (which is close to -1). It can be also seen that the coefficients of this processes will vary throughout time but are impossible to be estimated by just looking at the series. One can notice that similar conclusions can be drawn when looking at the raw TED spread data.

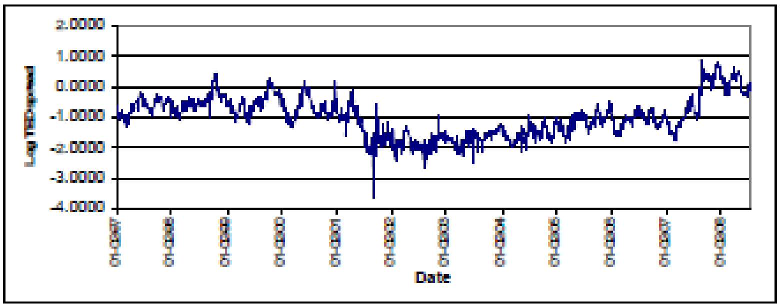

Fig. 2. Log TED spread (1997-2008)
Rys. 2. Log TED spread (1997-2008)
Source: own study.

Since it is suspected that data follow a mean reverting process characteristic, one can proceed to running the statistical tests. Following the methodology, first one should test whether the whole available series is mean reverting.

Table 1

Regression results, daily changes in the log the TED spread

	γ0	**γ1**	**t-stat γ0**	**t-stat γ1**	**R2**	**S.E. regression**
Log TED Spread (X)	–0.01926	–0.01926	–5.19315	–5.242199	0.009475	0.127058
p-value			(0.000)	(0.000)		
Augmented D-F		–3.12472				

Looking at the value of the coefficients as well as the t-statistic, it can be shown that γ_1 is significantly negative (99% confidence interval). Moreover, Dickey-Fuller test rejects the null hypothesis at the 95% confidence interval implying that the 10-year log spread does not follow a random walk process which, consequently, leads to the conclusion that it is mean reverting. When it comes to testing for a mean reverting process using data from the estimation period, the results are as follows in tab. 2

Table 2

Regression results, daily changes in the log of TED spread

	γ0	**γ1**	**t-stat γ0**	**t-stat γ1**	**R2**	**S.E. regression**
Log TED Spread (X)	–0.01935	–0.03415	–2.21359	–2.228168	0.016387	0.068989
p-value			0.0275	0.0266		
Augmented D-F		–2.64332				

Having in mind the previous reasoning, it can be deducted that the log TED spread from the estimation period follows a mean reverting process. It should be stressed, however, that the random walk hypothesis can be rejected at 90% confidence interval, indicating a bit weaker, but still significant proof of the mean reverting process. What is more, all estimated coefficients are statistically significant at a 95% confidence interval. One can notice that in both OLS regressions made, R_2 is very small, indicating that the regression does not describe the deviation from the sample mean (of changes in the log spread) very well.

However, coefficient values estimated using the ML method (in Matlab v 7.5) gave exactly the same values, increasing credibility of the obtained results. Since it can be shown that the data is mean reverting one can proceed to testing for homoscedasticity. It should be stressed that testing will be done using the log spread as well as risk-free rate data from the estimation period 03/14/2006 – 05/22/2007.

Looking at fig. 3, one can notice that the series is characterised by small variability within the estimation period. The logarithm of the TED spread oscillates within a narrow range of (-6, -5) implying that the data is homoscedastic.

Similar conclusions can be drawn depicting the behaviour of the riskless rate. The rate moves around a relatively tight range of (4.6%-5.2%) suggesting that the variance of the series is constant throughout the estimation period and implying that one can use the Vasicek's model to represent the behaviour of the riskless rate.

The cursory testing provides only the preliminary, and not very precise, proof that the data has a constant variance. One should now proceed to statistical testing to confirm or contradict the results from the initial analysis.

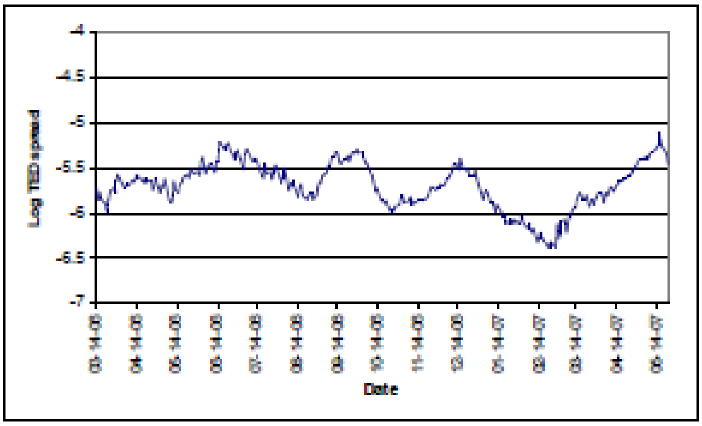

Fig. 3. Testing for homoscedasticity – The log TED spread Cursory Testing
Rys. 3. Testowanie homoscedasticity – The log TED spread Cursory Testing
Source: own study.

Table 3

ARCH LM test results – ARCH LM test

	Riskless rate r		**Log spread X**	
	F-stat	**N*R2**	**F-stat**	**N*R2**
Value	1.570268	30.2762	1.683287	13.26274
Probability	0.059942	0.0655	0.102138	0.103124

The data implies that the null hypothesis of no heteroscedasticity cannot be rejected at a 95% confidence interval for the riskless rate and a 90% confidence interval for the log spread. Hence, the ARCH LM test proves the observation made when eye-balling the data, leading to a final conclusion that both can be assumed to be homoscedastic. Last but not least, one should check the distribution of the log spread, to find the model that best fits the data.

Table 4

Summary statistics for the TED spread and changes in log spread

	Mean	**Std Dev**	**Kurtosis**	**Skewness**	**No of observations**
Spread (S)	0.003	0.001	2.513	0.2	300
Log Spread (X)	0.001	0.069	3.942	–0.086	300

The data shows that the Kurtosis coefficient of the log spread is a bit higher than in a normal distribution (it should be 3) and that there is a negative skewness. However, since the values do not differ from the normal distribution to a large extent, it is assumed that the log spread is approximately normally distributed.

Following steps stated in the previous section, from the analyses shown so far, one can conclude that it is appropriate to use the Longstaff and Schwartz's (1995) model. However, before applying the model to stand-by facility pricing, one has to first estimate the coefficients of the model.

It should be reminded that coefficients estimated in regression of the log spread changes have already been shown in the data when testing for the mean reverting process. Regression of the risk-free rate changes, in turn, as follows.

Table 5

Regression results, daily changes in the riskless rate

(03/14/2006-05/22/2007)

	ψ0	**ψ1**	**t-stat ψ0**	**t-stat ψ1**	**R^2**	**S.E. regression**
Riskless rate r	0.001185	–0.02359	2.591971	–2.571761	0.021784	0.000221
p-value			(0.01)	(0.01)		

It should be stressed that all coefficients presented in the data are statistically significant at the 99% confidence interval. As it was in the case of previous regressions, goodness of fit attains very small values. However, the coefficients obtained from ML estimation (in Matlab v 7.5) are of the same values as from the OLS, raising the credibility of the obtained results. Applying regression results to the set of formulas mentioned in the previous section, one obtains.

Table 6

Estimated parameters

Log TED spread (X)		**Riskless rate (r)**	
a =	–0.19124	α =	0.001185
b =	0.033825	β =	0.023876
s =	0.049417	Φ =	0.000158

Moreover, the correlation coefficient (ρ = **–0.90976)** shown below, implies that the riskless rate and the log TED spread react adversely to changes in one of the instruments. It is fully justifiable by the fact that the benchmark for the spread and risk-free rate consist of the same data. Construction of the TED spread implies that when the value of the spread increases, the distance from the riskless rate must increase as well. Having estimated model parameters one can apply them to the European spread call option pricing formula. Fig. 4 shows the sample spreadsheet to calculate the price of an at-the-money 1-year option.

	A	B	C	D	E
1	**European call option written on the TED spread**				
2					
3	**INPUT**				
4					
5	**estimated coefficients (X)**			**estimated coefficients (r)**	
6	**a=**	**-0.191242**		**α=**	**0.001185**
7	**b=**	**0.033825**		**β=**	**0.02387577**
8	**s=**	**0.049417**		**φ=**	**0.00015814**
9					
10	**ρ=**	**-0.909761**			
11	**r=**	**0.03**			
12	**K=**	**0.0035**			
13	**S=**	**0.0035**			
14	**T_D=**	**252**	(trading days)		
15	**T=**	**1**	(calendar years)		
16	**X=**	**-5.654992**	=LN(B13)		
17					
18	**OUTPUT**				
19					
20	**call option**	**0.000301**	=B30*EXP(B22+B25/2)*B28-B12*B30*B29		
21					
22	**μ=**	**-5.650287**	=(EXP(-B7*B14)*B16)+(1/B7*(B6-(B10*E8*B8)/E7))		
23			*(1-EXP(-B7*B14))+((B10*E8*B8)/(E7*(B7+E7)))*		
24			(1-EXP(-(B7+E7)*B14))		
25	**η^2=**	**0.036099**	=(B8^2*(1-EXP(-2*B7*B14)))/(2*B7)		
26	**d1=**	**0.214760**	=(-LN(B12)+B22+B25)/(B25^0.5)		
27	**d2=**	**0.024763**	=B26-B25^0.5		
28	**N(d1)=**	**0.585023**	=NORMSDIST(B26)		
29	**N(d2)=**	**0.509878**	=NORMSDIST(B27)		
30	**D(r,T)=**	**0.970446**	=EXP(-B14*B20)		
31	**Delta=**	**0.000115**	=B30*EXP(B22+B25/2)*B28*EXP(-B7*B14)*EXP(-		
32			B16)		

Fig. 4. European spread call option price spreadsheet
Rys. 4. Europejska opcja spread call w arkuszu ceny
Source: own study.

Changing the time to maturity and keeping other factors constant, one can calculate distribution of the option prices. It should be mentioned that both T_D and T (cells B14 and B15 respectively) should be changed when using different maturity options. Moreover, changing the TED spread value S (cell B13), immediately alters the log spread value X (cell B16) and enables one to price the option when it is in, at or out of the money. It should be mentioned that in the particular case shown of this data, K is equal to the mean of the spread *S*. Results of the call option price (cell B20), with different underlying levels and time to maturity are summarised.

Table 7

Sample prices of the CSO call option values of the spread and option price are in BPS

TED spread (S)	**T=1/12 (1 month)**	**T=3/12 (3 months)**	**T=1 (12 months)**
25	0.58	2.35	3.01
35	2.63	3.04	3.01
45	5.78	3.63	3.01

(Changing spread and time to maturity, other factors constant)

The data shows only certain option prices. The whole option payoff dependent on the underlying and time to maturity is shown in fig. 5. This graph shows features of the model that are consistent with observations made by Longstaff and Schwartz and Tahani[8]. One can notice that the option price may fall below the intrinsic value as the spread widens above 37 BPS. What is more, the option may have a negative convexity (as shown by the 3-month option). Above the certain time to maturity, the price of the option remains almost constant, independent of the underlying (as shown by the 1-year option). This behaviour is explained by the fact that with large values of the underlying and time to maturity it is expected that the spread will revert to its mean (which equal 35 BPS). One can notice on fig. 5 that given the estimated parameters, the price of the T = 1 option approaches a constant value much faster than observed by Longstaff and Schwartz, where the 1-year option was only becoming convex. This fact can be justified by the use of different data, and consequently, different estimated coefficients.

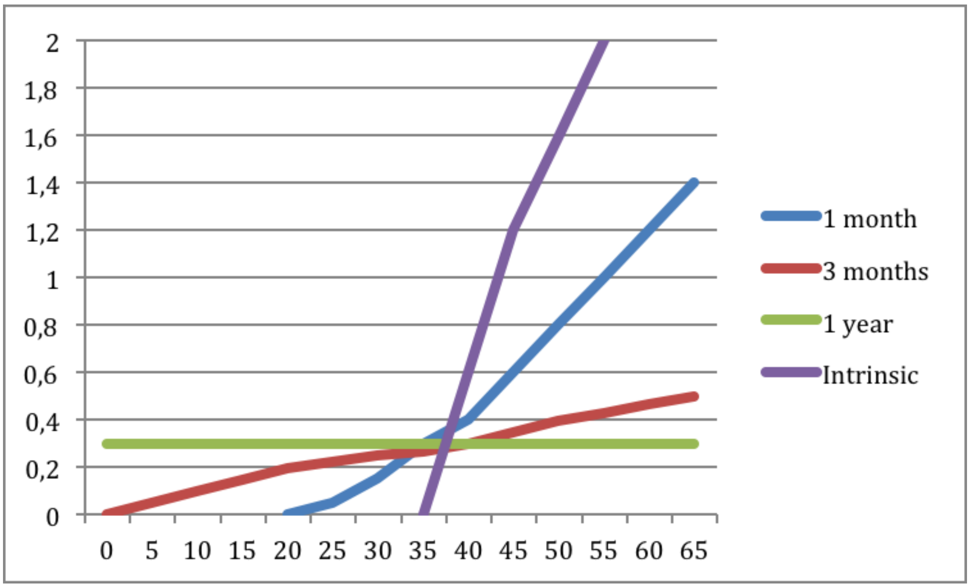

Fig. 5. European credit spread call option payoff (K = 35 BPS)
Rys. 5. Spread kredytowej opcji europejskiej wypłaty (K = 35 BPS)
Source: own study.

It may seem that in general the price of the option is very small. However, one should remember that the TED spread, or originally CP spread, is in BPS as it denotes the rate at which money will be borrowed. It is hard to compare the obtained prices with results obtained by other researchers. So far none of them used the TED spread as input data.

[8] Tahani N.: Credit Spread Option Valuation under Garch. Working Paper, July 2000, www.hec.ca/gestiondesrisques/papers.html.

4. Implications to stand-by facility pricing, conclusions and recommendations

As mentioned before, the proposed model might be used in a wide variety of applications. It is applicable to all types of ABCP programs mentioned in section 1, whether the fund provider is the conduit-issuing bank or the third party institution with a stand-by facility agreement. It should be stressed, however that the model can be used as a liquidity rather than credit enhancement facility. For simplicity, the discussion on pricing stand-by facilities will be based on a single-seller ABCP program. Further on, the SPV will be referred to as the fund borrower, and bank as the fund provider.

The SPV is a European call option buyer, as it has the opportunity to obtain funds from the bank when the option matures. The bank in, turn, is the European call option issuer as it is obliged to provide funds to troubled SPV, with whom it has a liquidity support agreement. One should remember that the calculated price of the option is equivalent to the possibility of borrowing 1£ from the fund provider at a given rate. Therefore, the obtained price should be multiplied by the amount of money that is going to be transferred in case of liquidity inadequacy. It is assumed that the SPV needs to borrow 1 bn £ to roll over issued Commercial Paper. For such a possibility it has to pay the bank the amount given:

Table 8

Price of the contingent claim per 1 bn £ (K = 35 BPS)

TED spread (S)	**T=1/12 (1 month)**	**T=3/12 (3 months)**	**T=1 (12 months)**
25	162 000	235 000	301 000
35	263 000	304 000	301 000
45	578 000	363 000	301 000

From the point of view of the bank, it is the sum that should be reported in the "off balance sheet activities" and taken into consideration together with Delta when planning for the future liquidity needs. Moreover, the price of the contingent claim should be taken into consideration in liquidity risk management of the bank because of regulatory issues. It should be stressed that due to lack of literature on the topic it is difficult to assess the obtained price of the contingent claim with respect to other agreements that the conduit might have with the fund providers.

The payoff of the European call option, from the point of view of the SPV is equivalent to the rate at which the conduit will borrow money from the bank compared to the rate at which it would borrow funds at given time from the other party, without initial agreement. Another very important implication is that for options with longer maturities (above 6 months), the price approaches a certain, almost constant, level irrespective of the value of the underlying, as depicted on the fig. 6.

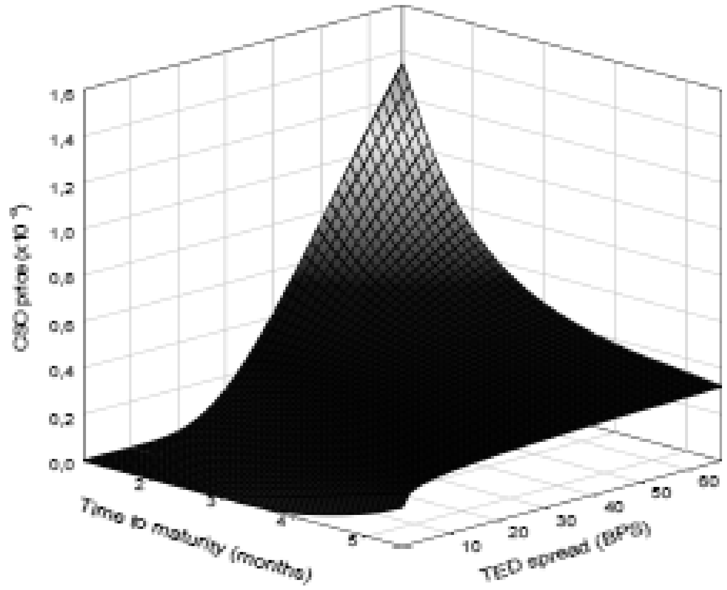

Fig. 6. European call option price with respect to changing T and S other parameters constant (K = 35 BPS)
Rys. 6. Cena europejskiej opcji ceny z wpływem na zmiany T i S, inne parametry stałe (K = 35 BPS)
Source: Own study.

The graph indicates that the price of the contingent claim will be the same for a large T, throughout different spread and strike levels. From the point of view of the bank, on one hand, it is a simplification in that the price of the claim will not change since it is independent of the level of the underlying (within a certain period of time). This in turn has a useful impact on medium term liquidity planning of the bank as well as on fulfilment of the regulatory requirements. On the other hand however, it should be stressed that as the option approaches maturity, its price might significantly change in value. This is especially well visible on fig. 6 for the deep in-the-money option (K = 65 BPS), as goes from T = 12 months down to T = 1 month, the price appreciates about 5 times. For a deep out-of-the-money option (K = 15) with the same time difference, the price of the contingent claim depreciates from 3 to almost 0 BPS. A bank's management should take into consideration that the price of the option will change from constant, towards a higher or lower value depending whether it is in- or out-of-the-money.

One can also observe that when the option is out-of-the-money, its value will be smaller, the lower the time to maturity. The situation is the opposite when pricing an option that is in-the-money (bearing in mind that the exercise level of K is at the level of the mean value of the

TED spread). The given data implies that differences of the contingent claim prices within various time intervals can be significant. The same table shows how significant changes in the balance sheet of the bank might appear as the option approaches maturity.

Table 9

Difference between various contingent claim prices, per 1 bn £

S (BPS)	1 year – 1 month	1 year – 3 months	3 months – 1 month
15	299 000	147 000	151 000
25	243 000	66 000	177 000
35	38 000	–3 000	41 000
45	–277 000	–62 000	–215 000
55	–628 000	–114 000	–514 000

From a regulatory point of view, it is recommended to impose restrictions on banks to monitor the price of options over short intervals, to avoid liquidity shocks when the option's time to maturity diminishes. Failure to do so, for example when the option is in-the-money (S = 55 BPS) and its maturity goes from 1 year to 1 month, might cause a significant jump in the option price and influence the liquidity of the bank.

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